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Analysis of Free Vibrations of Homogeneous Rectangular Thin Plates Using the Finite Difference Method (FDM)

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ABSTRACT

This study presents a numerical investigation of the free vibration behavior of homogeneous rectangular thin plates, formulated within the Kirchhoff–Love plate theory using the Finite Difference Method (FDM). Dimensionless natural frequencies for the first three vibration modes were computed under various classical boundary conditions—fully clamped (CCCC), supported (SSSS), and mixed (CSCS)—across multiple grid discretizations. The analysis focuses on the convergence of natural frequencies with grid refinement and the influence of boundary constraints on vibration characteristics. The proposed FDM framework employs central difference schemes for derivative approximations, ensuring high accuracy, numerical stability, and rapid convergence, with minimal change in computed frequencies beyond moderate grid sizes. Comparative results with existing studies confirm the approach's reliability and effectiveness. The findings reveal that boundary conditions significantly influence both mode shapes and frequencies. Fully clamped plates exhibit the greatest stiffness, producing the highest natural frequencies, while supported configurations yield lower frequency responses. Mixed boundary conditions produce intermediate behaviors, demonstrating the sensitivity of vibration characteristics to edge constraints. Overall, the findings provide essential insights into the structural design, optimization, and stability assessment of plate structures in engineering applications.

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INTRODUCTION

In modern engineering and structural design, rectangular plates are fundamental structural components widely used in various engineering systems. These plates play a critical role in load-bearing, support, and energy transmission in bridges, buildings, aircraft, ships, machinery, and other industrial equipment. To ensure reliable performance, accurate analysis of their free vibration behavior is essential. Free vibration refers to the condition in which a structure vibrates at its natural frequencies without external excitation. Understanding natural frequencies and corresponding mode shapes is crucial for maintaining structural integrity, extending service life, and preventing resonance-related failures. When the natural frequency of a plate coincides with external dynamic excitation frequencies, resonance may occur, potentially leading to severe damage or structural collapse. Researchers have developed various analytical and numerical techniques to study the dynamic behavior of rectangular plates, among which the Kirchhoff–Love plate theory remains one of the most widely adopted formulations for thin plates. This theory establishes the governing differential equation describing the vibrational response, enabling accurate determination of natural frequencies when solved appropriately. In the present study, the authors investigate the free vibration behavior of homogeneous rectangular thin plates using the Kirchhoff–Love theory. The study employs the Finite Difference Method (FDM) to obtain numerical solutions and uses central difference schemes to approximate derivatives. This approach allows for the systematic examination of variations in natural frequencies and deformation patterns across different mesh densities and boundary conditions.

The vibration problem of rectangular plates has attracted significant attention for more than two centuries. Their dynamic behavior is influenced by several parameters, including loading conditions, material properties, aspect ratio, and boundary constraints that define support conditions (Xing & Xu, 2013). Accurate determination of natural frequencies remains essential in civil, mechanical, naval, and aerospace engineering applications (Bakhshandeh et al., 2017). The earliest mathematical formulation of plate vibration problems dates back to Euler in 1776 (Ventsel & Krauthammer, 2001).

Wu et al. (2007) introduced a novel analytical technique employing the Bessel function to obtain exact solutions for the free vibration analysis of thin rectangular plates subjected to three specific boundary conditions: (1) completely supported, (2) completely clamped, and (3) two opposite edges supported with the remaining two edges clamped. This Bessel-function-based approach enables precise determination of both the natural frequencies and the mode shapes of rectangular plates. Researchers have also analyzed the free-vibration behavior of rectangular plates with mixed boundary conditions (clamped–supported) using the Superposition Method. This method relies on combining fundamental states and provides an accurate and efficient solution for complex boundary conditions. Gorman demonstrated that this technique serves as an effective alternative to Levy-type solutions for calculating natural frequencies and mode shapes(Gorman, 1977). Sakiyama and Huang (1998) also showed that variations in plate thickness produce significant changes in natural frequencies and mode shapes, which are of paramount importance for the effectiveness of plate design. Moreover, the vibration analysis of plates with variable thickness is more complex and challenging than that of uniform-thickness plates; however, such analyses are critically important in industrial and structural applications. Leissa (1973) reported exact and comprehensive theoretical findings on the free vibration behavior of rectangular plates under 21 different scenarios, including clamped, supported, and free-edge support configurations.

The Ordinary Finite Difference Method (OFDM) was applied to study the free vibration of thin rectangular plates. The study discretized the biharmonic equation into a finite-difference form over a grid, yielding algebraic equations that the analysis solved under various boundary conditions, including fully simply supported (SSSS), fully clamped (CCCC), and mixed clamped–simply supported (CSCS). The authors developed a Visual Basic program to perform the computations. The numerical natural frequencies closely matched exact solutions, demonstrating that OFDM is a simple and effective method for plate vibration analysis (Ezeh et al., 2013).

Additionally, the Finite Difference Method (FDM) was applied to solve the equations of motion for a spring-mass system, which represents a simple vibration model typically expressed as a second-order differential equation. In their investigation, the time domain was divided into small intervals, and numerical solutions were obtained using FDM. The study employed both explicit and implicit schemes and compared the results with analytical solutions. The findings show that the Finite Difference Method provides accurate and stable solutions, establishing it as a simple, efficient, and reliable method for analyzing basic vibration systems (Ningsi et al., 2020).

The free vibration of rectangular plates with central holes was investigated using the Finite Difference Method (FDM) because exact solutions are unavailable. Plates were discretized into grids, and the biharmonic equation was applied to interior and boundary points with various boundary conditions: fully clamped (CCCC), fully supported (SSSS), and clamped–supported (CSCS). Results showed that the hole's size and location significantly affect natural frequencies. This study confirms that FDM is a reliable and efficient numerical tool for analyzing vibrations of perforated rectangular plates (Hossain et al., 2015).

Sayyad and Ghugal (2015) provided an extensive review of free vibration analysis for laminated composite and sandwich plates. They covered physical properties, mechanical models, and a range of analytical, semi-analytical, and numerical methods, including FEM and FDM. The study presented numerical results and compared them with earlier research, concluding that suitable techniques can achieve accurate vibration analysis despite the complexity. Rezaei and Saidi (2019) developed an exact analytical solution for the free vibration analysis of thick rectangular plates made from porous materials. Using Reddy's third-order shear deformation theory, the study incorporated the effects of fluid saturation and porosity variations, along with various boundary conditions, to accurately determine the plates' natural frequencies and vibrational characteristics.

This study presents a precise numerical investigation of the free vibrations of thin, homogeneous rectangular plates using the Kirchhoff–Love plate theory. The Finite Difference Method (FDM) is applied to analyze variations in natural frequencies and mode shapes under different boundary conditions, including fully clamped (CCCC), fully simply supported (SSSS), and clamped–simply supported (CSCS) boundary conditions. Additionally, the study explores the effect of grid size on vibration responses to assess its impact on numerical result accuracy. The objectives of the study are as follows:

- To numerically examine the free vibration characteristics of thin, homogeneous rectangular plates.

- To study the bending response of plates based on the Kirchhoff–Love theory.
- To employ the Finite Difference Method (FDM) for solving the governing differential equations.
- To evaluate natural frequencies and vibration modes by discretizing the plate and using central difference approximations.

The present study differs from earlier investigations by providing a structured convergence analysis of the central difference finite difference scheme and by comparing its numerical performance with the forward difference formulation reported by Kumar (2018). This approach emphasizes grid-refinement behavior and numerical stability, providing a clearer evaluation of solution accuracy.

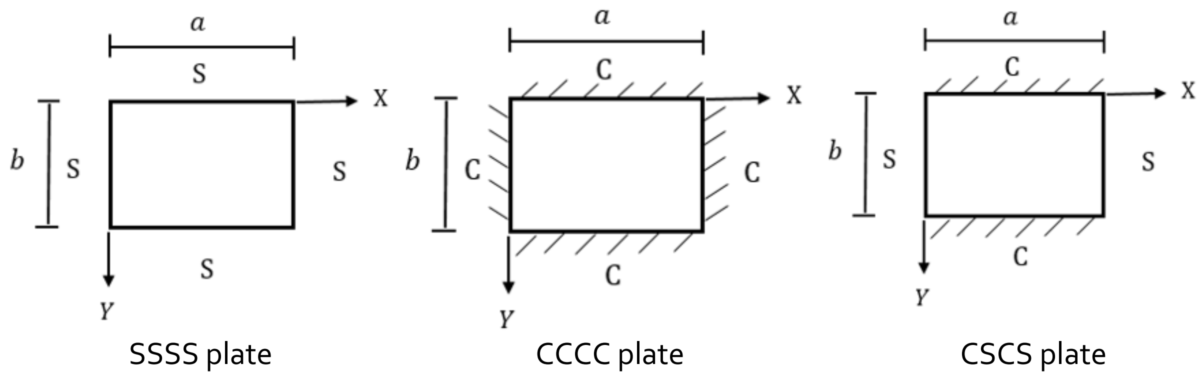

***Figure 1**. Illustrates the three types of boundary conditions*

METHODS AND MATERIALS

This research focuses on the numerical analysis of the free vibration behavior of homogeneous, thin, rectangular plates. The theoretical foundation underpinning the analysis is the Kirchhoff–Love plate theory, which describes the bending behavior of thin plates. The theory assumes that the plate thickness is significantly less than its length and breadth; therefore, it neglects shear deformation effects.

For the numerical solution of the fundamental differential equation, the study employed the Finite Difference Method (FDM). The analysis discretized the computational domain into a grid and approximated the differential terms in the governing equations using central differences.

The central difference approximation used in this study is second-order accurate, with a truncation error of order (h^2). Compared to the forward difference scheme $O(h)$, the central formulation provides improved accuracy for moderate grid sizes. Higher-order schemes such as $O(h^6)$ may yield better precision but require more complex stencil implementation.

Since the truncation error is of order $O(h^2)$, where h represents the grid spacing, refining the mesh improves the accuracy of the numerical solution. The stabilization of natural frequencies observed in Tables 5–7 confirms the method's convergence.

Methodological Steps

- Formulation of the governing equation based on the Kirchhoff–Love plate theory.
- Discretization and numerical solution of the equation using the FDM, with derivative approximations obtained via central difference.
- Implementation of various classical boundary conditions: CCCC, CSCS, and SSSS.
- Formulation and resolution of the eigenvalue problem for determining natural frequencies and mode shapes.
- All numerical computations and post-processing in this study were performed using MATLAB (R2023b, The MathWorks Inc., Natick, MA, USA).

FINDINGS

This section presents the numerical analysis of free vibrations of homogeneous rectangular plates using the Finite Difference Method. The study calculates natural frequencies for the first three modes under CCCC, SSSS, and CSCS boundary conditions. The results, tabulated for different grid sizes, show convergence and highlight the effect of boundary conditions on vibration behavior.

Plate Structure

A plate is a three-dimensional structural element whose thickness is much smaller than its other two dimensions. Under external loading, it primarily develops normal stresses, while shear stresses are typically negligible. Two parallel faces enclose the plate, and the perpendicular distance between these faces represents the plate's thickness (Ventsel & Krauthammer, 2001).

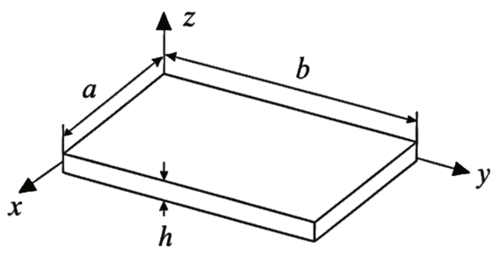

***Figure 2**. Rectangular plate and its coordinate system*

The two-dimensional (2D) plate theory provides approximate results that closely resemble the behavior of a three-dimensional (3D) plate structure. This condition arises because the plate's thickness is significantly smaller than its other two dimensions. Consequently, researchers assume the plate is in a plane-stress condition and that stresses through the thickness are zero (Kumar, 2018).

$$\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0, \tag{1}$$

Similarly, the shear strain components associated with the stress are also assumed to be zero:

$$\gamma_{xz} = \gamma_{yz} = 0. \tag{2}$$

Governing Differential Equation of a Homogeneous Rectangular Plate

According to the Kirchhoff–Love plate theory, when a plate is homogeneous—meaning that its material properties, such as Young's modulus E, density ρ, and thickness h, remain constant across the surface—the governing equation for its transverse vibration can be expressed as follows (Leissa, 1973; Ezeh et al., 2013):

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \tag{3}$$

Where $D = \frac{Eh^3}{12(1-v^2)}$ is flexural rigidity; w *is* transverse deflection, v is Poisson's ratio; and ∇^4 is the biharmonic differential operator, and t is time.
Applying the biharmonic operator to the transverse deflection yields:

$$\nabla^4(w) = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4},$$

By substituting the preceding relation into equation (3), we obtain:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h \frac{\partial^2 w}{\partial t^2} = 0. \tag{4}$$

The equation represents a fourth-order homogeneous partial differential equation.

However, because this study does not analyze time-dependent behavior and focuses only on $W(x, y)$, it employs the method of separation of variables to solve the equation. Using this method, the analysis separates the time-dependent component from the governing equation and reduces it to an eigenvalue problem (Ezeh et al., 2013).

$$D\nabla^4 W(x, y) = \rho h \omega^2 W(x, y). \tag{5}$$

The above equation is used to calculate the angular natural frequencies ω of free vibration and the $W(x, y)$ mode shapes. The mode shapes are independent of time and depend only on spatial variables, making them suitable for computational techniques such as the Finite Difference Method (FDM).

Finite Difference Method-Based Free Vibration Analysis of Rectangular Plates

To model the rectangular plate, the analysis divides the domain into equal $M \times N$ grid segments. Each grid cell has rectangular dimensions ΔX in the longitudinal direction and ΔY in the transverse direction. The grid points represent specific locations at which the numerical solution evaluates the variables. This discretization establishes a systematic computational framework for evaluating deformations and variations across different regions of the plate.

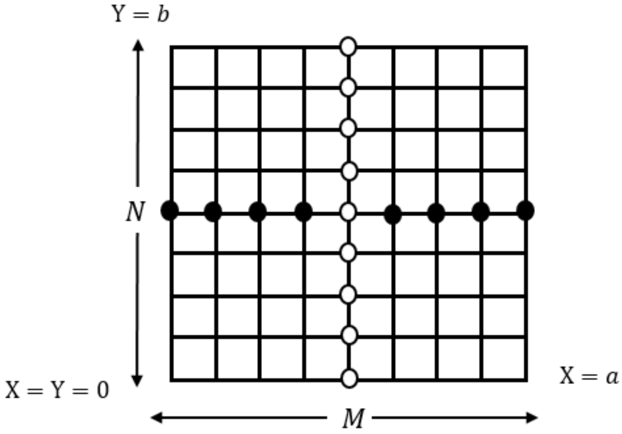

***Figure 3**. Rectangular Plate Surface Divided into a Regular Grid*

The analysis achieves this by drawing lines parallel to the X-axis and Y-axis, thereby forming a two-dimensional structured grid. The resulting grid consists of interior, corner, and boundary points used by the numerical analysis. The study defines the computational domain as follows:

$$0 \leq X \leq a \quad , \quad 0 \leq Y \leq b$$

That is, the geometric domain of the plate extends from $(0,0)$ to (a, b). The analysis distributes the total number of function values over $M \times N$ grid points in the computational domain, and it uses these points for the numerical analysis of the plate model. The grid is uniformly spaced, so all grid segments have equal lengths, a property that is essential for finite difference approximations in the FDM.

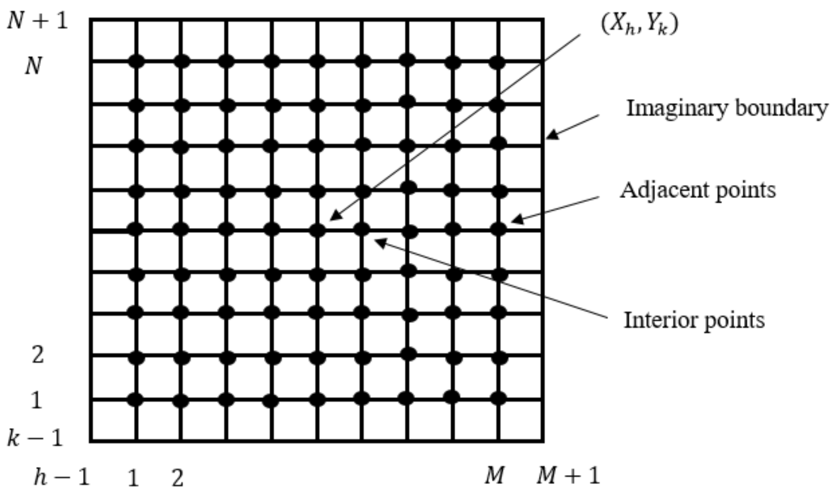

***Figure 4.** Interior Points, Adjacent Points, and Boundary of a Rectangular Plate*

In Figure 4, it is shown that the grid points are defined in the direction of the X-axis from $h-1$ to M and in the direction of the Y -axis from $k-1$ to N if a general grid point is denoted by(h,k), then the deflection or displacement at this point is represented by the function$W(h,k)$.

For homogeneous rectangular plates, the governing equation is solved numerically by means of the Finite Difference Method (FDM). The analysis employs the central difference scheme to approximate the derivatives. Based on this approximation, it formulates a finite-difference equation at each interior point and then assembles these equations into a matrix to enable numerical solution.

$$\left.\frac{\partial^4 w}{\partial X^4}\right|_{h,k} = \frac{w_{h-2,k} - 4w_{h-1,k} + 6w_{h,k} - 4w_{h+1,k} + w_{h+2,k}}{\Delta X^4}, \tag{6}$$

$$\left.\frac{\partial^4 w}{\partial Y^4}\right|_{h,k} = \frac{w_{h,k-2} - 4w_{h,k-1} + 6w_{h,k} - 4w_{h,k+1} + w_{h,k+2}}{\Delta Y^4}, \tag{7}$$

$$2\left.\frac{\partial^4 w}{\partial X^2 \partial Y^2}\right|_{h,k} = \frac{2[w_{h+1,k+1} + w_{h-1,k-1} + w_{h+1,k-1} + w_{h-1,k+1} - 2\left(w_{h+1,k} + w_{h-1,k} + w_{h,k+1} + w_{h,k-1}\right) + 4w_{h,k}]}{\Delta X^2 \Delta Y^2}. \tag{8}$$

Now, at each interior grid point(h,k), Equation (5) is expressed in its numerical form as follows:

$$D\left[\frac{\partial^4 w}{\partial X^4} + 2\frac{\partial^4 w}{\partial X^2 \partial Y^2} + \frac{\partial^4 w}{\partial Y^4}\right]_{h,k} = \rho h \omega^2 W(x,y)_{h,k}\,. \tag{9}$$

Boundary Conditions

For CCCC Case:

$$x = 0\,, (h = 0) \quad \wedge \quad x = a\,, (h = M+1), \tag{10}$$

$$w_{0,k} = 0\,, \quad w_{1,k} = 0\,, \quad w_{M,k} = 0\,, \quad w_{M+1,k} = 0, \tag{11}$$

$$\left.\frac{\partial w}{\partial x}\right|_{0,k} = \frac{w_{1,k} - w_{-1,k}}{2\Delta X} = 0 \Rightarrow w_{1,k} = w_{-1,k}, \tag{12}$$

$$\left.\frac{\partial w}{\partial x}\right|_{M+1,k} = \frac{w_{M+2,k} - w_{M,k}}{2\Delta X} = 0 \Rightarrow w_{M+2,k} = w_{M,k}, \tag{13}$$

$$y = 0\,, (k = 0) \quad \wedge \quad x = b\,, (k = n+1), \tag{14}$$

$$w_{h,0} = 0\,, \quad w_{h,1} = 0\,, \quad w_{h,N} = 0\,, \quad w_{h,N+1} = 0, \tag{15}$$

$$\left.\frac{\partial w}{\partial y}\right|_{h,0} = \frac{w_{h,0} - w_{h,-1}}{2\Delta Y} = 0 \Rightarrow w_{h,0} = w_{-h,0}\,, \tag{16}$$

$$\left.\frac{\partial w}{\partial y}\right|_{h,N+1} = \frac{w_{h,N+2} - w_{h,N}}{2\Delta Y} = 0 \Rightarrow w_{h,N+2} = w_{h,N}\,. \tag{17}$$

For SSSS Case:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{0,k} = \frac{w_{2,k} - 2w_{1,k} + w_{0,k}}{\Delta X^2} = 0 \;\Rightarrow w_{2,k} = 2w_{1,k}\,, \tag{18}$$

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{M+1,k} = \frac{w_{M+1,k} - 2w_{M,k} + w_{M-1,k}}{\Delta X^2} = 0 \;\Rightarrow w_{M-1,k} = 2w_{M,k}\,, \tag{19}$$

$$\left.\frac{\partial^2 w}{\partial y^2}\right|_{h,0} = \frac{w_{h,2} - 2w_{h,1} + w_{h,0}}{\Delta Y^2} = 0 \;\Rightarrow w_{h,2} = 2w_{h,1}\,, \tag{20}$$

$$\left.\frac{\partial^2 w}{\partial y^2}\right|_{h,N+1} = \frac{w_{h,N-1} - 2w_{h,N} + w_{h,N+1}}{\Delta Y^2} = 0 \;\Rightarrow w_{h,N-1} = 2w_{h,N}\,. \tag{21}$$

For CSCS Case:

Clamped at $(x = 0 \;\wedge\; x = a)$ on the x-axis

$$w_{0,k} = 0\;,\quad w_{1,k} = 0\;,\quad w_{M,k} = 0\;,\qquad w_{M+1,k} = 0\,, \tag{22}$$

$$\therefore\quad w_{1,k} = w_{-1,k}\quad,\qquad w_{M+2,k} = w_{M,k}\,, \tag{23}$$

Simply supported at $(y = 0 \;\wedge\; y = b)$ on the y-axi

$$w_{h,0} = 0\;,\quad w_{h,N+1} = 0, \tag{24}$$

$$\therefore\quad w_{h,2} = 2w_{h,1}\quad,\qquad w_{h,N-1} = 2w_{h,N}\,. \tag{25}$$

FDM Coefficients and Pattern

$\nabla^4 W$ is a biharmonic operator such that:

$$\frac{\partial^4 w}{\partial X^4} + 2\frac{\partial^4 w}{\partial X^2 \partial Y^2} + \frac{\partial^4 w}{\partial Y^4}, \tag{26}$$

For each interior grid point(h,k), the domain is discretized using central difference, and each boundary is expressed separately as follows:

For the $\boldsymbol{\frac{\partial^4 w}{\partial X^4}}$ term:

$$\left.\frac{\partial^4 w}{\partial X^4}\right|_{h,k} = \frac{w_{h-2,k} - 4w_{h-1,k} + 6w_{h,k} - 4w_{h+1,k} + w_{h+2,k}}{\Delta X^4}, \tag{27}$$

Table 1: Coefficients for the $\frac{\partial^4 w}{\partial X^4}$ term

Position	Coefficients
$(h \pm 2, k)$	$\frac{1}{\Delta X^4}$
(h, k)	$\frac{6}{\Delta X^4}$
$(h \pm 1, k)$	$-\frac{4}{\Delta X^4}$

For the $\frac{\partial^4 w}{\partial Y^4}$ term:

$$\left.\frac{\partial^4 w}{\partial Y^4}\right|_{h,k} = \frac{w_{h,k-2} - 4w_{h,k-1} + 6w_{h,k} - 4w_{h,k+1} + w_{h,k+2}}{\Delta Y^4}, \quad (28)$$

Table 2: Coefficients for the $\frac{\partial^4 w}{\partial Y^4}$ term

Position	Coefficients
$(h, k \pm 2)$	$\frac{1}{\Delta Y^4}$
(h, k)	$\frac{6}{\Delta Y^4}$
$(h, k \pm 1)$	$-\frac{4}{\Delta Y^4}$

For the $2\frac{\partial^4 w}{\partial X^2 \partial Y^2}$ term:

$$\left.2\frac{\partial^4 w}{\partial X^2 \partial Y^2}\right|_{h,k} = \frac{2[w_{h+1,k+1} + w_{h-1,k-1} + w_{h+1,k-1} + w_{h-1,k+1} - 2(w_{h+1,k} + w_{h-1,k} + w_{h,k+1} + w_{h,k-1}) + 4w_{h,k}]}{\Delta X^2 \Delta Y^2} \quad (29)$$

Table 3: Coefficients for the $2\frac{\partial^4 w}{\partial X^2 \partial Y^2}$ term

Position	Coefficients
$(h \pm 1, k \pm 1)$	$\frac{2}{\Delta X^2 \Delta Y^2}$
$(h \pm 1, k)$	$-\frac{4}{\Delta X^2 \Delta Y^2}$
$(h, k \pm 1)$	$-\frac{4}{\Delta X^2 \Delta Y^2}$
(h, k)	$\frac{8}{\Delta X^2 \Delta Y^2}$

Table 4: Coefficients for $\nabla^4 W$ at the (h, k) grid point are given as

Position	Coefficients
(h, k)	$\frac{6}{\Delta X^4} + \frac{6}{\Delta Y^4} + \frac{8}{\Delta X^2 \Delta Y^2}$
$(h \pm 1, k)$	$-\frac{4}{\Delta X^4} - \frac{4}{\Delta X^2 \Delta Y^2}$
$(h \pm 2, k)$	$\frac{1}{\Delta X^4}$
$(h, k \pm 1)$	$-\frac{4}{\Delta Y^4} - \frac{4}{\Delta X^2 \Delta Y^2}$
$(h, k \pm 2)$	$\frac{1}{\Delta Y^4}$
$(h \pm 1, k \pm 1)$	$\frac{2}{\Delta X^2 \Delta Y^2}$

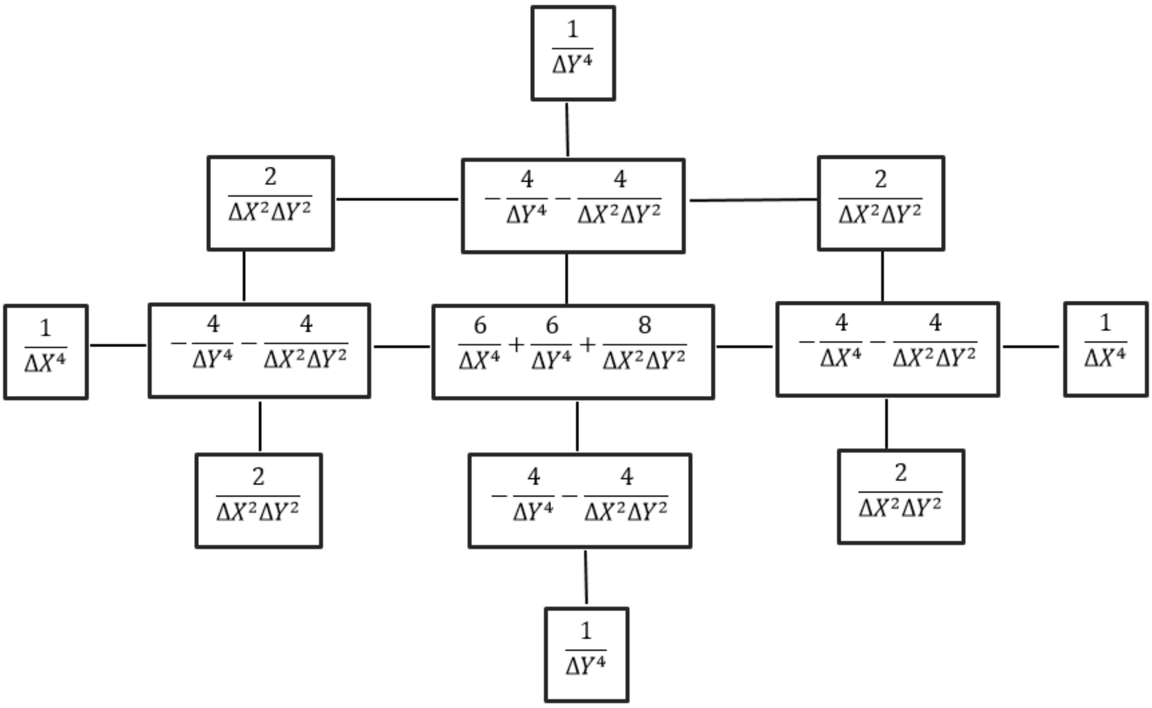

Figure 5. *Stencil Pattern for ∇^4 Operator*

Eigenvalue Equation

The eigenvalue equation is a fundamental mathematical formulation that yields the plate's natural frequencies and corresponding mode shapes. This information is essential for the analysis, design, and safe operation of the plate.

The eigenvalue equation derived from Equation (5) is expressed as follows:

$$(A - \lambda B)x = 0 , \quad (30)$$

Where $x = [x_i]$ is a column matrix whose elements represent the amplitudes of free vibration, $A = [a_{ij}]$ is a square matrix derived from the FDM of the biharmonic operator ∇^4, $B = [b_{ij}]$ is a diagonal matrix that represents a constant $\frac{\rho h}{D}$ and $\lambda = \omega^2$ is the angular or circular frequency.

Multiplying both sides of equation (30) by B^{-1} yields:

$$B^{-1}[A - \lambda B]x = 0, \tag{31}$$

$$[B^{-1}A - \lambda BB^{-1}]x = 0, \tag{32}$$

$$[B^{-1}A - \lambda I]x = 0, \tag{33}$$

Since I is the identity matrix in the above equation, letting $B^{-1}A = C$, we can rewrite it as:

$$[C - \lambda I]x = 0. \tag{34}$$

The resulting equation is the free eigenvalue equation for thin rectangular plates.

This equation has a non-trivial (non-zero) solution only when:

$$det[C - \lambda I] = 0, \tag{35}$$

$$\therefore\ |C - \lambda I| = \begin{bmatrix} c_{11-\lambda} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22-\lambda} & \cdots & c_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn-\lambda} \end{bmatrix} = 0, \tag{36}$$

This is called the characteristic equation. The values $\lambda_i (i = 1,2,3, \dots, n)$ denote the eigenvalues of the vibrating system, from which the angular frequencies $\omega_i = \sqrt{\lambda}, (i = 1,2,3, \dots, n)$ are determined (Ezeh et al., 2013).

Numerical Analysis

In this study, the natural frequencies of free vibrations of rectangular plates are expressed in non-dimensional form using the following relation (Y. Xing et al., 2022).

$$\overline{\omega} = \omega \cdot a^2 \cdot \sqrt{\frac{\rho h}{D}}, \tag{37}$$

Here, $\overline{\omega}$ denotes the non-dimensional natural frequency of the plate, ω is the angular frequency given by $\omega = \sqrt{\lambda}$ based on the eigenvalues λ and a, which represent the plate length.

This non-dimensionalization ensures that the results depend solely on the plate's geometric shape and boundary conditions, while remaining independent of its material properties, thickness, and overall dimensions. This approach enables accurate comparisons across different studies and strengthens the general validity of the findings.

Understanding the free vibration behavior of rectangular plates is essential for the design and stability assessment of mechanical structures. This study numerically analyzes the natural frequencies and mode shapes of homogeneous square plates with an aspect ratio of 1 using the Finite Difference Method (FDM), in which spatial derivatives are approximated using the central difference scheme under various boundary conditions. The central difference method is a robust numerical technique widely used to solve the biharmonic equations of plates. Its simple matrix formulation, ease of implementation, and the ability to refine grids to improve accuracy make it widely used in research. Natural frequency is the specific frequency at which a plate vibrates spontaneously without any external excitation. Each frequency is associated with a distinct vibration pattern or mode shape. These vibration modes play a vital role in nondestructive testing of structures and stability analysis.

In this analysis, the plate thickness and material properties, including Young's modulus(E), density(ρ), Poisson's ratio(ν), and the aspect ratio $\frac{a}{b} = 1$ are kept constant. The study varies only the grid size and the mode number. The numerical analysis is conducted using MATLAB, and for each plate, the computation considers only the first three smallest eigenvalues, corresponding to the first three natural frequencies. Employing the FDM approach, accurate natural frequency values are obtained for various grid sizes and modes of the square plate, providing reliable data for the design and analysis of mechanical systems.

Table 5: *Natural frequencies for* ***CCCC*** *boundary conditions when $a/b = 1$ and $\nu = 0.3$*

Mode No of Grid Points	I	II	III
M=N=4	35,020	71,240	71,255
M=N=6	34,991	72,110	72,125
M=N=8	34,982	72,360	72,370
M=N=10	34,980	72,395	72,395
M=N=12	34,980	72,398	72,398
M=N=14	34,980	72,398	72,398
M=N=16	34,980	72,398	72,398

Table 6: *Natural frequencies for* ***SSSS*** *boundary conditions when $a/b = 1$ and $\nu = 0.3$*

Mode No of Grid Points	I	II	III
M=N=4	27,975	53,950	65,039
M=N=6	27,960	53,930	68,199
M=N=8	27,951	53,910	68,320
M=N=10	27,950	53,908	68,327
M=N=12	27,950	53,907	68,327
M=N=14	27,950	53,907	68,327
M=N=16	27,950	53,907	68,327

Table 7: *Natural frequencies for* ***CSCS*** *boundary conditions when $a/b = 1$ and $\nu = 0.3$*

Mode No of Grid Points	I	II	III
M=N=4	28,121	54,115	65,520

M=N=6	27,970	53,930	68,310
M=N=8	27,956	53,920	68,327
M=N=10	27,951	53,910	68,327
M=N=12	27,951	53,910	68,327
M=N=14	27,951	53,910	68,327
M=N=16	27,951	53,910	68,327

DISCUSSION

In this study, the natural frequencies of free vibration of homogeneous rectangular plates were analyzed using the Finite Difference Method (FDM). The dimensionless fundamental frequencies of the first three vibration modes were computed for various grid sizes under the boundary conditions CCCC, SSSS, and CSCS.

The Finite Difference Method (FDM) is an effective numerical approach for solving the biharmonic equation, offering a straightforward matrix structure and easy implementation. This method shows that the grid size directly influences analysis accuracy, making it a fundamental requirement for the design and stability assessment of mechanical systems.

The tables clearly show that as the grid size increases, the natural frequency values stabilize. This convergence is a clear indication of the reliability and accuracy of the numerical solution obtained by the FDM using the central difference for derivative approximations.

Specifically, for the CCCC boundary conditions, the frequencies converge with minimal variation beyond a 10×10 grid size. Similar trends appear in the SSSS and CSCS cases, indicating that selecting an appropriate grid size is essential for obtaining accurate frequency calculations.

The study calculates and analyzes the first three mode shapes for each boundary condition. These mode shapes represent the plate's characteristic vibration patterns, providing essential information on its structural response and stability. The variation in mode shapes reflects differences in boundary conditions, illustrating the distinct deformation states of the plate. These results highlight the accuracy of the numerical analysis and demonstrate the broad applicability of the employed method.

The present study benchmarks its findings against Kumar’s (2018) research, which used the forward difference FDM, to evaluate the accuracy and stability of the Central Difference Method adopted here. In his study, Kumar calculated the natural frequencies of free vibrations of homogeneous rectangular plates under different boundary conditions (CCCC, CSCS) by increasing the grid size.

The comparison of the first three natural frequencies for the CCCC boundary condition shows that the results of this study are very close to Kumar’s findings. The results of both studies differ minimally, demonstrating the high accuracy of the CFDM. Specifically, at a grid size of ($M = N = 12$), the first natural frequency values in both studies are nearly identical at approximately 34.981. This indicates that the CFDM method provides fast, accurate results as the grid is refined.

For the CSCS boundary condition, both studies show good agreement. As the grid size increases, the natural frequencies stabilize, and both studies converge to similar values. In some cases, this study reports slightly higher frequencies than those presented by Kumar. This difference can be attributed to the distinct discretization schemes used in the central difference FDM and the forward difference FD.

Overall, this comparison demonstrates that FDM exhibits better convergence and higher accuracy than forward difference FDM. Additionally, it requires less grid refinement to achieve precise results. These findings demonstrate the reliability of the present study and its consistency with previous research.

***Table 8**: Comparison of the Present Study and Kumar's Research for **CCCC** Boundary Condition*

Mode # 1			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	34,982	34,9911	-0,026%
M=N=10	34,980	34,8815	0,282%
M=N=12	34,980	34,9815	-0,004%
M=N=14	34,980	34,9815	-0,004%
Mode # 2			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	72,360	71,0297	1,872%
M=N=10	72,395	72,2956	0,137%
M=N=12	72,398	72,3939	0,005%
M=N=14	72,398	72,3989	-0,001%
Mode # 3			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	72,370	71,0301	1,886%
M=N=10	72,395	72,2956	0,137%
M=N=12	72,398	72,3939	0,005%
M=N=14	72,398	72,3939	0,005%

***Table 9:** Comparison of the Present Study and Kumar's Research for **CSCS** Boundary Condition*

Mode # 1			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	27,956	27,5662	1,41%
M=N=10	27,951	27,9560	-0,01%
M=N=12	27,951	27,9509	0%

M=N=14	27,951	27,9509	0%
Mode # 2			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	53,920	53,6742	0,45%
M=N=10	53,910	52,7323	2,23%
M=N=12	53,910	53,7431	0,31%
M=N=14	53,910	53,7431	0,31%
Mode # 3			
Grid Size (MxN)	Present Study	Kumar (2018)	Difference %
M=N=8	68,327	65,0396	5,05%
M=N=10	68,327	68,1993	0,18%
M=N=12	68,327	68,3270	0%
M=N=14	68,327	68,3270	0%

The above tables compare the dimensionless natural frequencies obtained in this study using the Central Difference Finite Difference Method (FDM) with the results reported by Kumar (2018) for the first three vibration modes of a homogeneous rectangular plate under CCCC and CSCS boundary conditions. The comparison indicates that the present results are in close agreement with those of Kumar, with only minor discrepancies observed. For the first mode, the differences are minimal. In contrast, for the second and third modes, the deviations become progressively smaller as the grid size increases, confirming the accuracy, stability, and robustness of the proposed method. Slight variations appear for smaller grids (e.g.,$M = N = 8$), where the present study predicts marginally higher frequencies. These differences can be attributed to factors such as boundary condition implementation, the FDM scheme's sensitivity, and grid resolution. Overall, the results confirm that the central difference FDM yields reliable and precise solutions, with accuracy improving significantly as the grid size is refined.

For smaller grids such as $M = N = 4$ and $M = N = 8$, the results show larger deviations due to discretization error. As the grid is refined, the error decreases significantly, which is consistent with the second-order accuracy of the central difference scheme. The reduced accuracy at coarse grids, therefore, reflects expected numerical behavior.

This analysis shows that the FDM method, with derivatives computed using central differences, is highly accurate for the analysis of plate-free vibrations, particularly for higher modes. Moreover, proper implementation of boundary conditions is crucial for validating the reliability of the results.

Limitations

The present study is limited to thin homogeneous plates based on Kirchhoff–Love theory, in which shear deformation is neglected. The formulation is restricted to uniform structured grids and classical boundary conditions. Extending the approach to thick plates, non-homogeneous materials, or irregular geometries requires further investigation.

CONCLUSION

In this research, the dimensionless natural frequencies of free vibrations in homogeneous rectangular plates were determined using the Finite Difference Method (FDM) based on the Kirchhoff–Love plate theory. The study aimed to provide a precise numerical analysis of the vibrational characteristics of plates and to examine the influence of different boundary conditions. The results showed that as the grid size increased, the computed frequencies converged, confirming the method's stability and validity.

It was also observed that boundary conditions (CCCC, SSSS, and CSCS) strongly affect the vibrational behavior of plates. The degree of edge restraint alters the stiffness, which directly influences the natural frequencies and mode shapes. Among them, the CCCC condition produced the highest values due to full clamping of all edges, while SSSS and CSCS introduced more flexibility, leading to lower frequencies.

The study compares the computed results with those reported by Kumar (2018) and finds good agreement, demonstrating that the Finite Difference Method is both accurate and efficient, requiring less grid refinement and reduced computation time. These results highlight its value as a reliable numerical technique for plate vibration analysis.

In conclusion, the study successfully achieved its objectives by providing meaningful insights into the vibrational characteristics of rectangular plates. The findings establish a strong foundation for further research and offer practical guidelines for engineering applications. The results confirm that the natural frequencies follow the order $CCCC > CSCS > SSSS$ and that the frequency values converge with increasing grid size, closely matching Kumar (2018).

AUTHORS CONTRIBUTIONS

- Noorullah Noori conceptualized and supervised the study.
- Ghaniullah Safi and Noorullah Noori conducted the field study and collected the raw data.
- Ghaniullah Safi, Noorullah Noori, and Abdul Wakil Baidar equally processed and analyzed the data.

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CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

DATA AVAILABILITY STATEMENT

Data are available from the corresponding author upon reasonable request and subject to approval by the relevant ethics committee.

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