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Comparison of Newton Raphson – Linear Theory and Hardy Cross Methods Calculations for a Looped Water Supply Network

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ABSTRACT

This study conducts a comparative analysis between the Newton-Raphson and Hardy Cross methods for solving a looped main linear water network consisting of 4 pipes. The research findings demonstrate a high degree of unity between the outcomes obtained from these two methods, thereby validating their accuracy and reliability in solving water network equations. While the Newton-Raphson method shows faster convergence than the Hardy-Cross Method, both approaches effectively plan and analyze water networks. The analytical methodology employed in this study provides valuable insights into the applicability and efficiency of these methods in optimizing gravity main water networks. By combining the strengths of the Newton-Raphson and Hardy Cross methods, engineers and planners can make informed decisions to enhance the performance and sustainability of water distribution systems. The findings contribute to advancements in water infrastructure planning and design, aiming to ensure efficient and reliable water supply to meet the evolving needs of urban and rural communities.

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Introduction

Distribution networks are a fundamental portion of all water supply systems. Analysis of a pipe network is vital to understanding or evaluating a physical system, thus making it an integral part of the synthesis process of a network (Ackers, 1958). Water pipe networks play a crucial role in supplying fresh water from sources to end-users, with a significant portion of the total water supply system cost allocated to transmission and distribution networks. In the realm of hydraulic engineering, the analysis of complex water networks involves solving nonlinear equations that govern internal flow in pipes (Sultanbekova et al., 2021).

To tackle this challenge, numerical methods such as the Hardy-Cross method and the Newton-Raphson method are widely employed (Michele, 2012). These methods solve intricate equations arising from considerations like network continuity, Bernoulli's principle, and significant head loss equations. The primary objective of a water distribution network is to supply water at adequate pressures and flow rates within urban areas while ensuring potable water quality for consumers (Alan, 2000).

One of the earliest methods for determining the distribution of water flow and pressure in a water supply network is the renowned Hardy Cross method, an iterative technique for calculating flow in pipe networks where the inflows and outflows are known. Still, the internal flow is unknown (Sultanbekova et al., 2021). The Hardy Cross method is an adaptation of the Moment Distribution method, which Hardy Cross also developed to determine moments in indeterminate structures. Introduced by American engineer Hardy Cross in 1936, the Hardy Cross method has been extensively used to analyze pipe network flow (Campbell, 1993). These interconnected pipelines form a "water distribution network" that can adopt a branched, looped, or hybrid topological configuration. To analyze the steady-state behavior, the hydraulic network flows must be balanced. For branched networks, where the flow direction is evident, determining the flow rate in each conduit becomes straightforward once the flow demand at each terminal point is known (Gupta et al., 2010). This method revolutionized how engineers approached complex hydraulic and structural engineering problems long before the advent of modern computing technology (Basica et al., 2003).

By simplifying the intricate task of solving numerous equations in network analysis, Hardy Cross paved the way for a more efficient and systematic approach to designing and optimizing pipe systems (Bruce *et al*., 2009).

Through a comprehensive analysis of the Hardy Cross method and its variants, we aim to shed light on the enduring impact of Hardy Cross's pioneering work pipe network analysis. By understanding the principles and applications of this method, engineers can effectively tackle the challenges posed by looped piping systems and optimize the design and operation of complex networks (Cross, 1936).

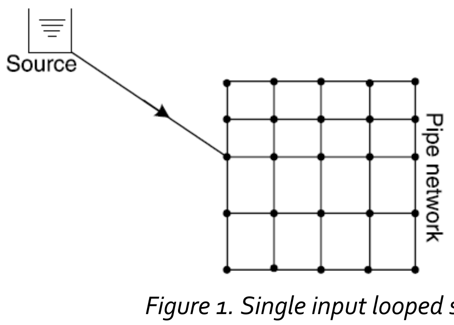


Figure 1. Single input looped system

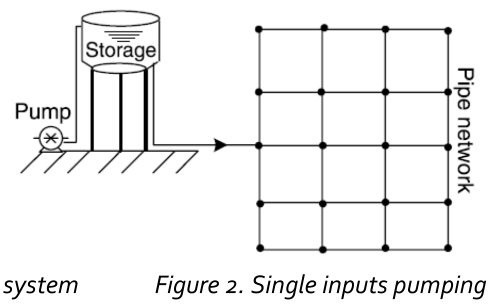


Figure 2. Single inputs pumping system

The looped network systems provide repetition to the systems, which increases the capacity of the system to overcome the nearby variation in water needs and ensures water distribution to consumers in case of pipe failures. (Doe, 2005). The looped geometry is additionally favored from the water quality aspect because it would diminish the water age. The strategy was afterward made out of date by computer-fathoming algorithms utilizing Newton-Raphson or other arrangement strategies that removed the requirement to comprehend the nonlinear framework of conditions by hand. Water distribution networks serve numerous purposes besides the arrangement of water for human utilization, which regularly accounts for less than 2% of the entire volume. The plot of one looped network is shown in Figure 3 **(**Darvishi *et al*., 2007).

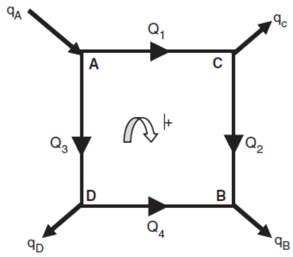


Figure3.Plot of one-loop network

The primary function of a pipeline network is to facilitate the transportation of fluids from production sites to consumption points while ensuring adequate pressure and flow rates. The conveyance of fluids over long distances is a significant concern for manufacturers. Extensive research has demonstrated that the most efficient method for fluid transportation is through visible or underground pipeline systems (Giustolisi, 2010).

These interconnected pipelines form a "water distribution network" that can adopt a branched, looped, or hybrid topological configuration. It is essential to establish a balanced flow distribution to analyze the steady-state hydraulic behavior of such networks. For branched networks, where the flow direction is evident, determining the flow rate in each conduit becomes straightforward once the flow demand at each terminal point is known (Gupta et al., 2010).

The hydraulic and optimization analyses are linked through an iterative procedure. The pipe network analysis estimates the discharge in each pipe, velocities, and the system's total cost. Also, proof of the solution in each method and the comparison between the two will be considered. The paper develops a nonlinear calculation for the optimal design of looped networks supplied by direct pumping from one or more node sources.

Three distinct procedures exist for analyzing the flow and pressure distribution in looped water supply systems: the loop method, the node method, and the branch method, each involving the selection of hydraulic parameters as unknowns (Hund et al., 2008). Regardless of the method chosen, the resulting nonlinear system of equations can be solved through iterative techniques, such as the Hardy–Cross method and the Newton–Raphson method. This paper presents a unique approach to these issues by employing various formulations for the functional analysis of water distribution systems. The study uses an Enhanced Hardy Cross Method to explore the Hydraulic Analysis of Water Supply Networks. The research aims to improve the solution process for hydraulic analysis in water distribution systems, thereby enhancing efficiency and accuracy in network analysis and design.

The proposed algorithm for selecting initial discharges aims to streamline the convergence process and reduce computational effort, ultimately leading to more precise results in water supply network analysis (Morris, 1988). The study's findings emphasize the faster convergence of the Newton-Raphson method compared to the Hardy-Cross Method, indicating potential efficiency gains in network analysis. To be more precise, the objectives of the study are as follows:

- To provide a precise method, it has been simple and reliable for planning and designing looped water supply networks.
- To explore the use of Hardy-Cross-linear theory and Newton-Raphson's method in calculating looped water supply networks.

Methods & Materials

Using analytical research, we examined the components and relationships of Newton-Raphson, linear theory, and hardy-cross methods for a looped water supply network. There are many methods for calculating circular water supply networks, among which three famous methods have been selected. One of the most common vital steps in water resources engineering is pipe network analysis; the essential techniques for this analysis are Hardy Cross, Newton-Raphson, and Linear Theory.

The study selects the Newton Raphson and Hard Darcy methods as the numerical techniques to solve the nonlinear algebraic equations derived from the water network's characteristics. These methods are chosen for their established effectiveness in handling complex systems of equations in hydraulic engineering. The data generated from the application of both methods is analyzed to draw comparisons and insights regarding the performance of the Newton Raphson and Hard Darcy methods in solving the nonlinear equations governing the gravity-leading water network.

Findings and Discussion

Overall, the study's findings provide valuable insights for engineers and water resource professionals in selecting the most suitable method for analyzing water supply networks based on system complexity, computational efficiency, and accuracy requirements. The study contributes to advancing the understanding and optimization of water infrastructure planning and design processes by presenting a detailed comparison of these methods and their applicability in different network scenarios.

Newton Raphson Method: Original approach

Newton-Raphson's method is one of the most powerful numerical methods to calculate nonlinear equations (Sarwar *et al*., 2002).

Assume we have three nonlinear equations:

$$F1=(Q1,Q2,Q3)=0 \quad , F2=(Q1,Q2,Q3)=0 , F3=(Q1,Q2,Q3)=0$$

Each should be calculated for the current Q1, Q2, and Q3 values.

Assume that the prices Q1+Q1, Q2+Q2, and Q3+Q3 solve the above equations

$$F1 (Q1+\Delta Q1 , Q2+\Delta Q2 ,Q3+\Delta Q3) =0$$

$$F2 (Q1+\Delta Q1 , Q2+\Delta Q2 ,Q3+\Delta Q3) =0$$

$$F3 (Q1+\Delta Q1 , Q2+\Delta Q2 ,Q3+\Delta Q3) =0$$

By developing the above equations in the form of the Taylor series, we have that:

$$F1+\left[\frac{\partial F1}{\partial Q1}\right]\Delta Q1+\left[\frac{\partial F1}{\partial Q2}\right]\Delta Q2+\left[\frac{\partial F1}{\partial Q3}\right]\Delta Q3=0$$

$$F2+\left[\frac{\partial F2}{\partial Q1}\right]\Delta Q1+\left[\frac{\partial F2}{\partial Q2}\right]\Delta Q2+\left[\frac{\partial F2}{\partial Q3}\right]\Delta Q3=0$$

$$F3+\left[\frac{\partial F3}{\partial Q1}\right]\Delta Q1+\left[\frac{\partial F3}{\partial Q2}\right]\Delta Q2+\left[\frac{\partial F3}{\partial Q3}\right]\Delta Q3=0$$

We arrange the set of the above equations in matrix form.

$$\begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix} \begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = - \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

After solving this equation, we have:

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = - \begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

The corrected current values are obtained as follows:

$$\begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix} = \begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix} + \begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix}$$

The following sections explain all of the steps for analysis using the Newton-Raphson method:

1. Grading of all nodes, pipes, and rings.
2. The equation describes the flow in each node. $\mathrm{Fj} = \sum_{n=0}^{Jn} \mathrm{Qjn} - \mathrm{qj} = 0$
3. We write the ring pressure loss equation $Fk = \sum_{n=1}^{Kn} Kn\ Qkn[Qkn]$
4. We take the initial flow value in the pipes hypothetically......Q1, Q2, Q3
5. We assume the price of f is equal to 0.02 and get the price of k by the following equation" $K = \frac{8\mathrm{FL}}{\pi 2\mathrm{gD5}}$
6. We obtain the price of partial derivatives, $\frac{\partial F1}{\partial Q1}$ and the function Fn using the assumed values and Ki.
7. The price $\Delta Q1$ comes in the form of Ax=b
8. After obtaining the price of $\Delta Q1$, this price is added to the assumed flow value, and the corrected flow value is received; we continue this process until we get the real flow value in the pipes.

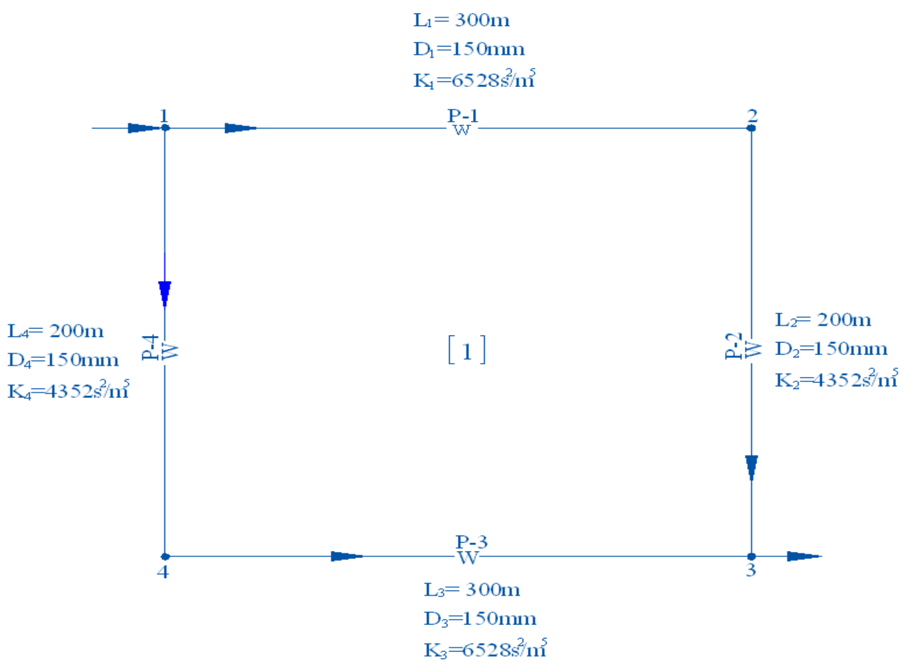


Figure4.Computational scheme of a circular network by Newton Raphson Method

The F function of the current value in the node is calculated as follows:

$$F1 = Q1 + \ Q4 \ - 0.6 = 0$$

$$F2 = -Q1 + \ Q2 \ = 0$$

$$F3 = Q2 + \ Q3 \ - 0.6 = 0$$

The pressure loss function is as follows:

$$F4 = K1[Q1]Q1 + K2[Q2]Q2 - K3[Q3]Q3 - K4[Q4]Q4$$

$$F4 = 6528[Q1]Q1 + 4352[Q2]Q2 - 6528[Q3]Q3 - 4352[Q4]Q4$$

We calculate the partial derivatives of the above equations and write the following:

$$\frac{\partial F_1}{\partial Q_1} = 1 \qquad \frac{\partial F_1}{\partial Q_2} = 0 \qquad \frac{\partial F_1}{\partial Q_3} = 0 \qquad \frac{\partial F_1}{\partial Q_4} = 1$$

$$\frac{\partial F_2}{\partial Q_1} = -1 \qquad \frac{\partial F_2}{\partial Q_2} = 1 \qquad \frac{\partial F_2}{\partial Q_3} = 0 \qquad \frac{\partial F_2}{\partial Q_4} = 0$$

$$\frac{\partial F_3}{\partial Q_1} = 0 \qquad \frac{\partial F_3}{\partial Q_2} = 1 \qquad \frac{\partial F_3}{\partial Q_3} = 1 \qquad \frac{\partial F_3}{\partial Q_4} = 0$$

$$\frac{\partial F_4}{\partial Q_1} = 6528Q_1 \qquad \frac{\partial F_4}{\partial Q_2} = 4352Q_2 \qquad \frac{\partial F_4}{\partial Q_3} = -6528Q_3 \qquad \frac{\partial F_4}{\partial Q_4} = -4352Q_4$$

We write the equations obtained above in the form of a matrix below:

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial Q_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial Q_3} & \frac{\partial F_1}{\partial Q_4} \\ \frac{\partial F_2}{\partial Q_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial Q_3} & \frac{\partial F_2}{\partial Q_4} \\ \frac{\partial F_3}{\partial Q_1} & \frac{\partial F_3}{\partial Q_2} & \frac{\partial F_3}{\partial Q_3} & \frac{\partial F_3}{\partial Q_4} \\ \frac{\partial F_4}{\partial Q_1} & \frac{\partial F_4}{\partial Q_2} & \frac{\partial F_4}{\partial Q_3} & \frac{\partial F_4}{\partial Q_4} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \end{bmatrix}$$

After applying the above derivatives, the following figure is obtained:

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 6528\,Q_1 & 4352Q_2 & -6528\,Q_3 & -4352Q_4 \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \end{bmatrix}$$

Assuming the amount of current in the tap is (Q_1=0.5m3) and in the other taps is based on the continuity equations, the current values are obtained as follows:

$$Q_{2=0.5m^3}$$

$$Q_{3=0.1m^3}$$

$$Q_{4=0.1m^3}$$

By putting these prices into the above equation, the following figure is obtained:

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 3264 & 2176 & -652.8 & -435.2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2611.2 \end{bmatrix}$$

Using the Gaussian method, solving the above equation gives the following results:

$$\Delta Q_{1=-0.2m^3/s}$$

$$\Delta Q_{2=-0.2m^3/s}$$

$$\Delta Q_{3=0.2m^3/s}$$

$$\Delta Q_{4=0.2m^3/s}$$

Using the corrected values, the corrected current values are obtained as follows:

$$Q_1 = Q_1 + \Delta Q_{1=0.5m^3/s-0.2m^3/s=0.3m^3/s}$$
$$Q_2 = Q_2 + \Delta Q_{2=0.5m^3/s-0.2m^3/s=0.3m^3/s}$$
$$Q_3 = Q_3 + \Delta Q_{3=0.1m^3/s+0.2m^3/s=0.3m^3/s}$$
$$Q_4 = Q_4 + \Delta Q_{4=0.1m^3/s+0.2m^3/s=0.3m^3/s}$$

By setting the modified prices, the following solution is obtained:

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1958.4 & 1305.6 & -1958.4 & -1305.6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

After simplifying the above matrix, $\Delta Q1 = 0$ is obtained, and we can write the actual current values as follows:

$$Q_{1=0.3m^3/s}$$
$$Q_{2=0.3m^3/s}$$
$$Q_{3=0.3m^3/s}$$
$$Q_{4=0.3m^3/s}$$

Linear Theory Method

Wood and Charles devised the linear theory method for analyzing the water supply network system in 1972. (Martin *et al.*, 1963). The whole network is calculated together like the Newton-Raphson method. The equation of the flow values in the nodes is linear, but the equation of the losses in the ring is nonlinear. The equation of the losses in the ring is changed to a linear form and calculated in a circular form, which continues (Michele *et al.*, 2012). He finds that until the solution set of both equations reaches an acceptable number, the equation of the flow values in the nodes is as follows:

$$\mathrm{F_j} = \sum_{n=0}^{Jn} \mathrm{Q_{jn}} - \mathrm{q_j} = 0$$

The above equation is generally written for the whole network as follows:

$$\mathrm{F_j} = \sum_{n=0}^{Jn} \mathrm{a_{jn}Q_{jn}} - \mathrm{q_j} = 0$$

The pressure loss equation in this system is written as follows:

$$\mathrm{F_k} = \sum_{n=0}^{Kn} \mathrm{K_n[Q_{Kn}]} - \mathrm{Q_{Kn}} = 0$$

The above equation can be linearized as follows:

$$\mathrm{F_k} = \sum_{n=0}^{Kn} \mathrm{b_{kn}Q_{Kn}} = 0$$

The analysis process of circular networks is based on the linear theory method as follows:

We score the pipes, nodes, and loops and write the equations of the current values in each node.

$$F_j = \sum_{n=0}^{Jn} Q_{jn} - q_j = 0$$

The ring pressure loss equation:

$$F_k = \sum_{n=0}^{Kn} b_{kn} Q_{Kn} = 0$$

Assuming the initial flow values, i.e., Q1, Q2, and Q3, it is unnecessary to fit the equation of the principle of continuity. Assume the fraction factor f=0.02, calculate the price k, and make node continuity and circular equations for the entire network. The values of the existing pipes are calculated. The resulting equation is Ab=x, which may be solved for 1Q. We repeat the process until the price q1, computed using two consecutive corrections, is within a previously defined range.

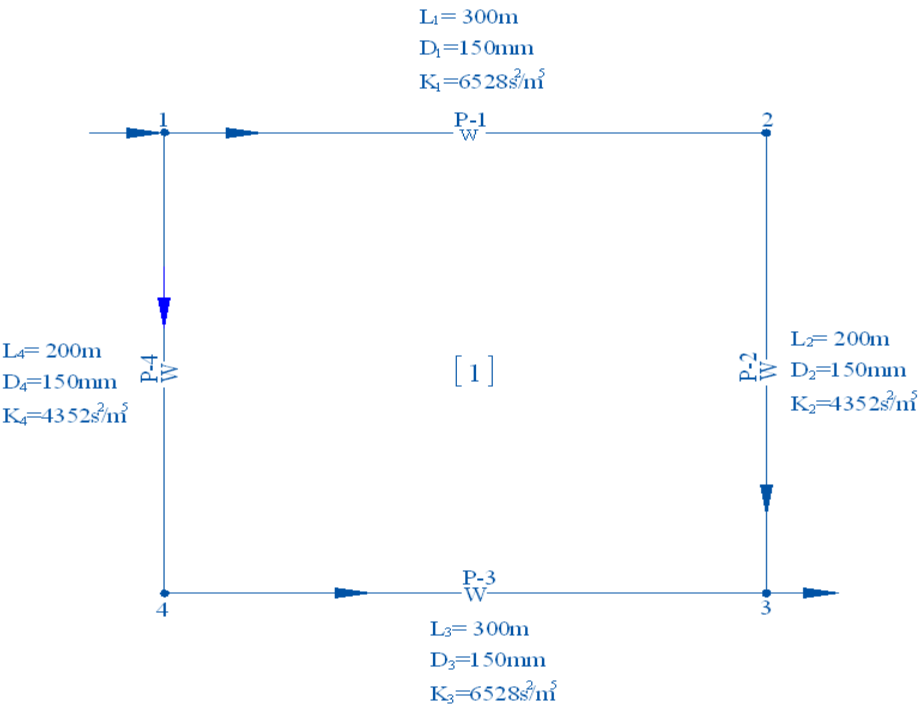


Figure5.Computational scheme of a circular network by linear theory

The F function of the flow discharge in each node is calculated as follows:

$$F1 = Q1 + \ Q4 \ - 0.6 = 0$$

$$F2 = -Q1 + \ Q2 \ = 0$$

$$F3 = Q2 + \ Q3 \ - 0.6 = 0$$

The pressure loss function is written below:

$$F4 = K1[Q1]Q1 + K2[Q2]Q2 - K3[Q3]Q3 - K4[Q4]Q4$$

The above equation is converted into a linear equation as follows:

$$F_4 = b_1Q_1 + b_2Q_2 - b_3Q_3 - b_4Q_4$$

The initial flow values are assumed to be 0.1m3/s in each pipe, and the pressure loss function coefficient is calculated as follows:

$$b_1 = K_1Q_1 = 6528 * 0.3 = 1958.4$$
$$b_2 = K_2Q_2 = 4352 * 0.3 = 1305.6$$
$$b_3 = K_3Q_3 = 6528 * 0.3 = 1958.4$$
$$b_4 = K_4Q_4 = 4352 * 0.3 = 1305.3$$

Now, the matrix of the form Ab=x can be written.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 652.8 & 435.2 & -6528.8 & -435.2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = - \begin{bmatrix} 0.6 \\ 0 \\ 0.6 \\ 0 \end{bmatrix}$$

By solving the above set of linear equations, the values of the currents in the pipes are obtained.

$$Q_{1=0.3m^3/s}$$
$$Q_{2=0.3m^3/s}$$
$$Q_{3=0.3m^3/s}$$
$$Q_{4=0.3m^3/s}$$

By repeating the process, the modified loss coefficient is written below.

$$b_1 = K_1Q_1 = 6528 * 0.3 = 1958.4$$
$$b_2 = K_2Q_2 = 4352 * 0.3 = 1305.6$$
$$b_3 = K_3Q_3 = 6528 * 0.1 = 1958.4$$
$$b_4 = K_4Q_4 = 4352 * 0.1 = 1305.6$$

Now, in the matrix shape, we can write b = xA.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1958.4 & 1305.6 & -1958.4 & -1305.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = - \begin{bmatrix} 0.6 \\ 0 \\ 0.6 \\ 0 \end{bmatrix}$$

By solving the above set of linear equations, the current values obtained are as follows:

$$Q_{1=0.3m^3/s}$$
$$Q_{2=0.3m^3/s}$$
$$Q_{3=0.3m^3/s}$$
$$Q_{4=0.3m^3/s}$$

Hardy Cross Method

The Hardy Cross method, proposed by Hardy Cross in 1936, has been a fundamental tool in analyzing flow in pipe networks (Jaefarzadeh *et al.*, 2014). This method revolutionized engineering practices by simplifying the complex calculations required for hydraulic and structural analysis long before the era of advanced computational tools. In this research, we explore the historical development and significance of the Hardy Cross method, tracing its evolution from its inception to the present day. We investigate the contributions of other researchers who have built upon Hardy Cross's work, such as Lobačev and Andrijašev, and examine the modifications and enhancements made to the original method by Epp, Fowler, Hamam, Brameller, Wood, Charles, Wood, Rayes, Shamir, and Howard. By analyzing these advancements, we highlight the continued relevance and application of the Hardy Cross method in contemporary engineering practice. Through a comprehensive review of the literature and case studies, we demonstrate the practical utility of the Hardy Cross method in addressing the challenges posed by looped piping systems. By understanding the principles and applications of this method, engineers can optimize the design, operation, and maintenance of pipe networks, ensuring efficient and reliable performance in various engineering applications (Sharma, 1992).

$$h_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} = h_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + h_L$$

1. Kinetic Energy (v2/2g)
2. Potential Energy (h)
3. Pressure Energy (p/γ)

The Hardy Cross method is extensively taught in academic institutions and remains essential for analyzing looped pipe systems. Contemporary engineers predominantly utilize a modified version of the Hardy Cross method, which considers the entire looped network of pipes simultaneously (using these methods without computational assistance is practically infeasible).

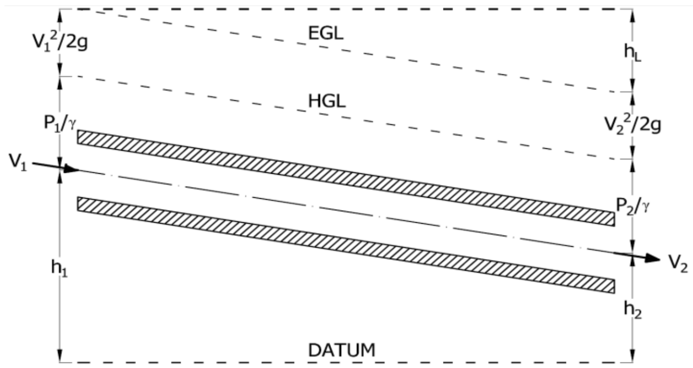


Figure 6. Energy Head and Head Loss in Pressurized Pipe Flow

Where V = Mean velocity; g = Gravity; h = Elevation; P = Pressure and γ = Specific weight of water

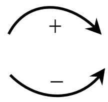
In the Hardy Cross method, a water flow correction value is added to or subtracted from the assumed flow values in each calculation (Sultanbekova *et al.*, 2021). This process continues until the correction current's price equals or near zero. The Hardy Cross method for calculating the hydraulic parameters of the circular water distribution network is summarized below:

1. The amount and direction of water flow in each faucet are calculated hypothetically. It should be noted that the amount of incoming current to a node equals the amount of outgoing current from the same node.

2. The Hazen-Williams formula calculates the pressure loss due to the friction effect of drain water flow. The water flow, which is clockwise, is assumed to be positive, and the opposite is negative.

$$H_L = \frac{1}{0.094}\left(\frac{Q_a}{100}\right)^{1.85}\left(\frac{L}{D^{4.87}}\right), m$$

3. In the above formula, Q is the amount of water flow in cubic meters per second, C is the coarseness coefficient of the faucet, which is taken for PE 130 faucets, and D is the diameter of the pipe in meters.
4. The corrected current value for the ring network is received and added to the hypothetical current value.
5. This process continues until the value of the correction constant becomes zero or close to zero.
6. In this method, the clockwise direction of water flow is considered positive, and the anti-clockwise direction is considered negative.



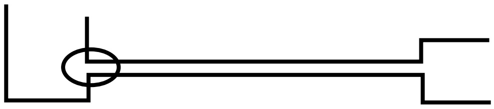
The corrected discharge value was described in Table 1 below with the following iterations.

Table1. corrected value of discharge

1	2	3	4			5			6		7	8
Test Number	Ring Nubmer	Pipe N°	Diameter by inch	Direction	Q (m^3/sec)	K (s^2/m^5)	KQ|Q|(m)	∑KQ|Q|(m)	2K|Q|(s/m^2)	∑2K|Q|(s/m2)	∆Q	Qnew (m^3/sec)
1	1	1		1	0.1000	6528.9300	65.29	-2611.57	1305.79	13057.86	0.200	0.3000
		2		1	0.1000	4352.6200	43.53		870.52		0.200	0.3000
		3		-1	0.5000	6528.9300	-1632.23		6528.93		0.200	-0.3000
		4		-1	0.5000	4352.6200	-1088.16		4352.62		0.200	-0.3000
2	1	1		1	0.3000	6528.9300	587.60	0.00	3917.36	13057.86	0.000	0.3000
		2		1	0.3000	4352.6200	391.74		2611.57		0.000	0.3000
		3		-1	0.3000	6528.9300	-587.60		3917.36		0.000	-0.3000
		4		-1	0.3000	4352.6200	-391.74		2611.57		0.000	-0.3000

The minor losses in pipe

In a typical piping system, the fluid encounters various obstructions such as fittings, valves, bends, elbows, tees, junctions, exits, expansions, and contractions, in addition to the pipes themselves. These components impede the smooth flow of the fluid and cause additional losses due to the flow separation and mixing they induce. In a typical system with long pipes, these losses are minor compared to the overall head loss within the pipes (the major losses) and are referred to as minor losses (Sarwar et al., 2002).



$$h_L = k \cdot \frac{v^2}{2g}$$

Figure 7: The minor head losses in the pipe

Where hl = minor losses; k= loss coefficient; v= internal velocity and g = gravity acceleration

The unit head loss

The unit head loss due to friction was examined to understand better the effects of pipe size and roughness (independent of pipe length). Although the implications of roughness and size on the head loss relationship may seem apparent, it is worthwhile to consider pipe classification (i.e., pipe class) based on distinct combinations of pipe size and roughness. The unit head loss using the Hazen-Williams relationship is represented in Eq.

$$H_f = \frac{1}{0.094} \times \left(\frac{Q}{C}\right)^{1.85} \times \frac{L}{D^{4.87}} \quad , \quad m$$

Where Q = pipe flow, C = roughness coefficient of the pipe, and D = internal diameter of the pipe.

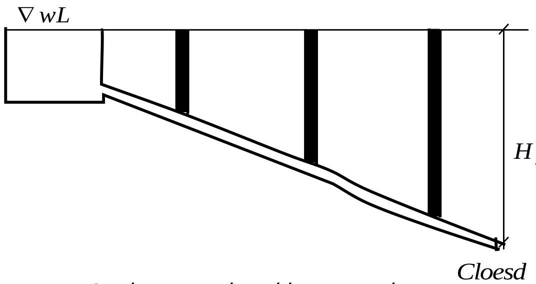


Figure 8. The minor head losses in the pipe

Still, the pressure losses along the pipe can be obtained using Darcy's relation.

$$H_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g}$$

Where f = friction factor; L = Length of the pipe; D = internal diameter of the pipe; v = velocity and g = Gravity acceleration

Implications for the future development of water supply infrastructure in urban areas. The implications of this study for the future development of water supply infrastructure in urban areas are significant. By comparing the Newtono-Raphson and Hardy Cross methods for analyzing water networks, this research provides valuable insights to guide the planning and design of more efficient and reliable water distribution systems. The findings suggest that leveraging advanced numerical techniques like the Newton-Raphson method can lead to faster convergence and improved efficiency in network analysis. This can be particularly beneficial for urban areas where water demand is high and pressure loss in distribution networks needs to be minimized (Charles *et al*., 1972). By understanding the strengths and limitations of different analysis methods, engineers and planners can make informed decisions to optimize the design and operation of gravity main water networks in urban settings (Maleki *et al*., 2016). Ultimately, the research contributes to enhancing the performance and sustainability of water supply infrastructure in urban areas, ensuring that communities have access to safe and reliable water resources now and in the future.

Conclusions

In conclusion, the research comparing the Newton-Raphson-linear theory and Hardy Cross methods for looped Water supply Networks highlights the significance of utilizing advanced numerical techniques in optimizing water distribution systems. The study's findings highlight the Newton-Raphson method's faster convergence compared to the Hardy Cross Method, indicating possible efficiency advantages in network research. The remarkable level of agreement between the findings of both methods demonstrates their accuracy and utility in solving complex water network equations. By leveraging the strengths of these methods, engineers, and planners can enhance the design and operation of gravity main water networks, paving the way for more efficient and sustainable water supply infrastructure in urban and rural areas. In conclusion, understanding these differences can help engineers and researchers choose the most appropriate method for analyzing water supply networks based on system complexity, computational efficiency, and accuracy requirements.

This study reviewed, scrutinized, and presented the Newton-Raphson – linear theory and Hardy Cross approaches in matrix format. One disadvantage of the Hardy-Cross approach is that it requires establishing continuity equations at the start of analysis to prevent incorrect initial values from being selected. Guessing can hinder convergence or even cause divergence. A method for choosing the initial guess was presented to address this issue. The variable initial guess solutions are near the ultimate solution and satisfy the continuity equations. Using this strategy at the beginning reduces the number of convergence iterations in the Hardy-Cross method. This approach was integrated with third- and sixteenth-order algorithms to accelerate analysis. The combined algorithms resulted in less iteration.

Conflict of Interests

The author declares that they have no conflicts of interest to disclose.

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Availability of data and materials

All data used during the study are available from the corresponding author by request.

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